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SHORTER COMMUNICATION

APPLICATION OF THE DEFECT LAW TO THE DETERMINATION OF THE AVERAGE VELOCITY AND TEMPERATURE IN TURBULENT PIPE FLOW*

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NOMENCLATURE

u, velocity;
r, radial co-ordinate;
r_0, radius of tube;
y, distance from wall;
$f(\ )$, function of;
T, temperature;
A, total Prandtl number;
D, a constant;
k, Nikuradze roughness parameter;
ν, kinematic viscosity;
κ, thermal diffusivity;
ϵ_M, eddy viscosity;
ϵ_H, eddy diffusivity;
λ, friction coefficient;
Δ, limit of velocity defect function as $r \to r_0$.

Dimensionless groups
Re, Reynolds number based on diameter and average velocity;
Pr, Prandtl number.

Subscripts
c, value at axis;
av, average value, for r_0 value at which local value equals average;
δ, value at edge of sublayer;
τ, friction.

INTRODUCTION

THE problem of determining the average velocity or temperature of a fluid flowing turbulently through a round pipe by a measurement at a single point is of some practical importance because of the inconvenience of making traverses or using mixing chambers.

Recently Rogers and Mayhew [1] analyzed two methods using the universal velocity and temperature distribution laws for turbulent flow in smooth pipes. The axial method uses the velocity or temperature on the axis and a correction factor which is a function of Reynolds number for the velocity and of Reynolds and Prandtl number for temperature. The ordinate method is based on measuring the velocity or temperature at a point where its value is equal to the average value.

This note considers the applications of defect profiles to these methods. While we are also limited to the restriction of essentially isothermal flow so that there is no marked variation of fluid properties across the profile, our results for the ordinate method are also applicable to rough pipes.

BACKGROUND ON TURBULENT FLOW

It is well known that the velocity profiles for both smooth and rough pipe are described quite well by the defect law

$$\frac{u_c - u}{u_\tau} = f(r/r_0). \tag{1}$$

The best known expression is the familiar logarithmic law

$$\frac{u_c - u}{u_\tau} = 5{\cdot}5 \log_{10} \frac{r_0}{r - r_0}. \tag{2}$$

We will refer to expressions such as this which become infinite as $r \to r_0$ as divergent. For parts of the present analysis it is preferable to have an expression for the velocity defect which approaches a finite value as $r \to r_0$. This type of expression will be referred to as convergent. The expression [2]

$$\left(\frac{r}{r_0}\right)^2 = 0{\cdot}189 \frac{u_c - u}{u_\tau} \exp\left(0{\cdot}0695 \frac{u_c - u}{u_\tau}\right) \tag{3}$$

is used for specific calculations, but the general method can be used with any expression.

A convergent expression is inconsistent with the usual assumption that the junction between the turbulent core and the laminar sublayer occurs at a fixed value of u_δ/u_τ as this would imply a constant friction coefficient. On the basis of Nikuradze's data, the relation

$$\frac{u_\delta}{u_\tau} = 5{\cdot}25 \log_{10} (Re \times 10^{-3}) \tag{4a}$$

was obtained [2]. For fully developed roughness it was found that

$$\frac{u_\delta}{u_\tau} = 5{\cdot}85 \log_{10} \left(\frac{r_0}{k} \times 10^{-1}\right). \tag{4b}$$

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The author [2, 3] has suggested that there is a relation between the temperature and velocity profiles of the form

$$\frac{T_c - T}{T_\tau} = A(Re, Pr)\frac{u_c - u}{u_\tau} \tag{5}$$

where the friction temperature T_τ is

$$\frac{\kappa}{u_\tau}\left.\frac{dT}{dr}\right|_{r=r_0}.$$

The proportionality factor A can be interpreted as a total Prandtl number, i.e. the ratio of the sum of the eddy and molecular viscosities to the sum of the eddy and molecular diffusivities. An expression

$$A = \frac{1{\cdot}41\, Re\, Pr\, \sqrt{\lambda} + 40 Pr}{1{\cdot}20\, Re\, Pr\, \sqrt{\lambda} + 40} \tag{6}$$

was proposed for A. While the direct experimental evidence is incomplete, particularly for $A \ll 1$, the derivation of a heat transfer relation in good agreement with experiment in the liquid metal range is an indirect verification.

There are two types of average temperature in the literature. The theorists prefer the average defined by

$$\frac{\int_0^{r_0} r\, T\, dr}{\int_0^{r_0} r\, dr}$$

while the experimentalists prefer the bulk average defined by

$$\frac{\int_0^{r_0} r\, u\, T\, dr}{\int_0^{r_0} r\, u\, dr},$$

since it is easier to measure. The difference is small and usually neglected.

THE ORDINATE METHOD

It is obvious that if the defect law is assumed the ordinate at which the local velocity is equal to the average velocity is independent of Reynolds number or roughness. This is related to the well-known Prandtl rule that

$$\frac{u_c - u_{av}}{u_\tau} = D \cong 4{\cdot}07. \tag{7}$$

To obtain the value of the ordinate it is necessary to solve

$$D = f_1\left(\frac{r_{av}}{r_0}\right). \tag{8}$$

Either a divergent or convergent velocity distribution may be used. Using expression (3) gives $r_{av} = 0{\cdot}76\, r_0$ as the ordinate for the velocity determination. This agrees closely with Jakob [4] who claimed that a measurement at $0{\cdot}77\, r_0$ would determine the average velocity within 1 per cent. The variation obtained by Rogers and Mayhew is also relatively small going from $0{\cdot}808\, r_0$ at $Re = 10^4$ to $0{\cdot}777\, r_0$ at $Re = 10^7$.

Similarly if we assume the temperature defect law, equation (5), it can be seen that the ordinate at which the temperature is equal to the average temperature is the same as for the velocity determination, irrespective of the exact form of the expression for A. The evaluation of the ordinate for determining the bulk average temperature is more difficult and requires specific functional forms for the distributions. The ordinate will be a function of Reynolds and Prandtl number so that convenience is lost.

THE AXIAL METHOD

The simplification of the analytical results for the ordinate method obtained in the previous section does not change the fact that it uses a measurement at a point where the gradient is appreciable, while the axial method uses a measurement less subject to positioning error. While the calculation of the correction factor requires more specific assumptions, they are simpler than the corresponding assumptions utilizing the universal law.

To compute the ratio of average to maximum velocity, it is necessary to use a convergent (in the sense of this paper) velocity distribution. If we designate the limit of $(u_c - u)/u_\tau$ by Δ we have

$$\frac{u_{av}}{u_c} = 1 - \frac{D}{(u_\delta/u_\tau) + \Delta}. \tag{9}$$

In conjunction with (4) this gives

$$\frac{u_{av}}{u_c} = 1 - \frac{4{\cdot}07}{14{\cdot}4 + 5{\cdot}25 \log_{10} Re \times 10^{-3}} \tag{10}$$

for smooth pipes. In Fig. 1 this correction factor is compared with these proposed by Rogers and Mayhew and by Jakob. Some of Nikuradze's experimental points, as tabulated by Bates [5] are shown for comparison. It should, however, be noted that there is some dispute [6] about the data reduction.

In conjunction with equation (5) the relation

$$\frac{u_{av}}{u_c} = 1 - \frac{4{\cdot}07}{14{\cdot}4 + 5{\cdot}85 \log_{10} (r_0/k) \times 10^{-1}} \tag{11}$$

is obtained for rough pipes. In Fig. 2 this is compared with some of Nikuradze's experimental data [7].

The derivation of the correction factor for the average temperature requires an explicit expression such as (6) for the total Prandtl number and an expression for T_δ/T_τ. In [3] the relation

$$\frac{T_\delta}{T_\tau} = Pr\frac{u_\delta}{u_\tau} \tag{12}$$

was proposed, but it is limited to smooth pipes and may not be valid for large Pr. However, the analysis employed by Rogers and Mayhew is in the same category in regard to behavior at large Pr. The analog of (10) is

$$\frac{T_{av}}{T_c} = 1 - \frac{4{\cdot}07}{14{\cdot}4 + 5{\cdot}25\,\dfrac{1 + 0{\cdot}030\, Re\, Pr\, \sqrt{\lambda}}{1 + 0{\cdot}035\, Re\, \sqrt{\lambda}} \log_{10} Re \times 10^{-3}} \tag{13}$$

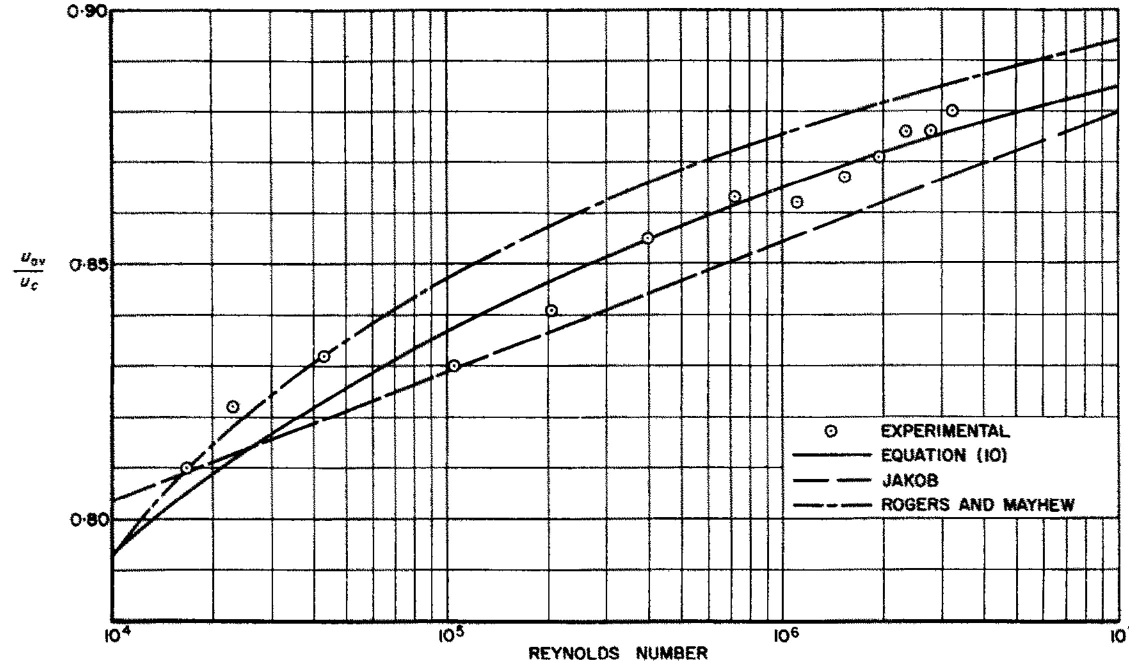

FIG. 1. Correction factor for axial method of average velocity determination in smooth pipe.

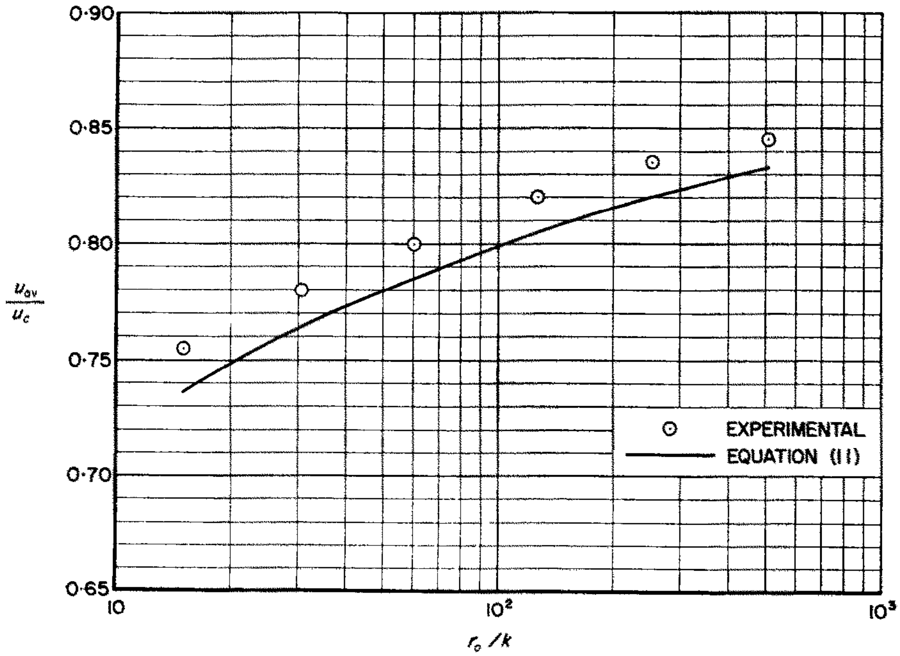

FIG. 2. Correction factor for axial method of average velocity determination in fully rough pipe.

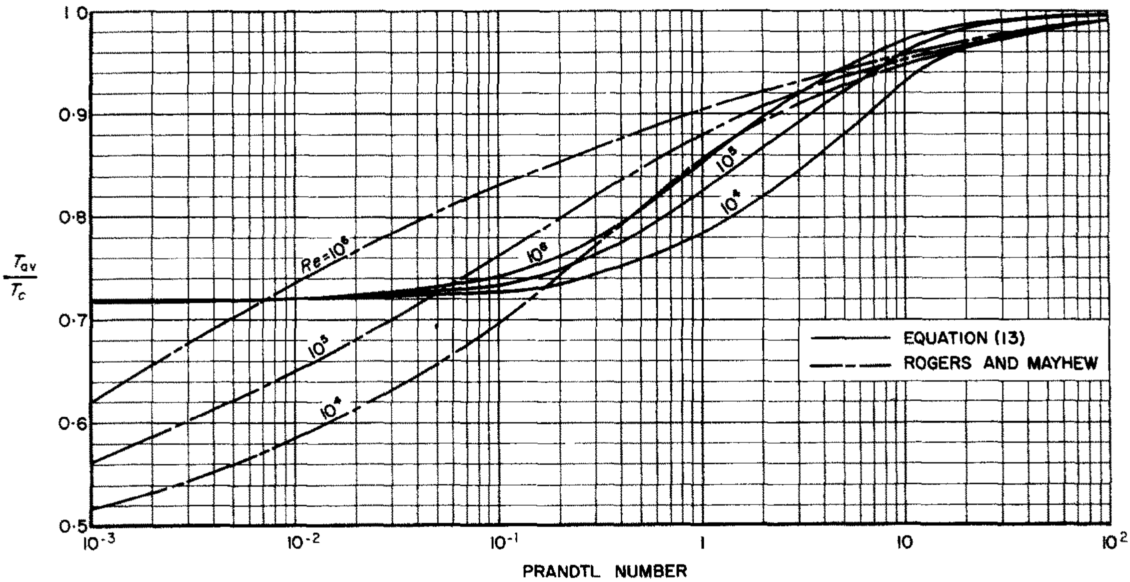

FIG. 3. Correction factor for axial method of average temperature determination in smooth pipe.

which is plotted in Fig. 3 with Mayhew and Rogers' factor for the bulk average. The calculation of the bulk average can be carried out by the present analysis but requires a specific expression for $f(r/r_0)$. It is believed that the difference between the average and bulk average is not large enough to account for the difference between the two analyses.

DISCUSSION

We have shown that the use of the defect laws results in an appreciable simplification of the analysis, but gives results somewhat different from those obtained using the universal laws. Actually, the defect law and the universal law are not exact mathematical relations but approximations of limited accuracy. For example, a critical examination of Nikuradze's data by Robertson [8] indicates a systematic difference in the value of D between smooth and rough pipes and a slight Reynolds number dependence. This may explain the small systematic discrepancy between the theoretical and experimental points in Fig. 2.

An advantage of our formulation of the ordinate method is that since the location is fixed the method should be applicable to cases where there is a slow time variation.

There is obviously a need for further experimental work, particularly on the temperature distribution, to resolve the discrepancies between the analysis based on the defect law and the analysis based on the universal profiles. In particular, closely spaced measurements in the neighborhood of the ordinate where the local value is equal to the average value are needed.

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